Luther-Emery Stripes, RVB Spin Liquid Background and High T_c Superconductivity

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The stripe phase in high T_c cuprates is modeled as a single stripe coupled to the RVB spin liquid background by the single particle hopping process. In normal state, the strong pairing correlation inherent in RVB state is thus transfered into the Luttinger stripe and drives it toward spin-gap formation described by Luther-Emery Model. The establishment of global phase coherence in superconducting state contributes to a more relevant coupling to Luther-Emery Stripe and leads to gap opening in both spin and charge sectors. Physical consequences of the present picture are discussed, and emphasis is put on the unification of different energy scales relevant to cuprates, and good agreement is found with the available experimental results, especially in ARPES.

The universal presence of phase separation in high T_c cuprates has been confirmed by extensive experiments, including elastic and inelastic neutron scatterings [1], NMR and NQR [2], and Angular Resolved Photon Emission Spectroscopy [3](ARPES) in $La_{2-x}Sr_xCuO_4$, $YBa_2Cu_3O_{7-x}$ and $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi-2212) etc. The emerging picture is , upon hole doping beyond $x = 0.06$, quarter-filled [4] hole rich stripes begin to form, separating the copper oxide plane into slices of antiferromagnetic insulating regions, with the inter-stripe distance in proportion to $1/x$, where x is the density of doped holes. Above $x = 1/8$ and inside the overdoped regime, incommensurate stripe modulation persists, although the inter-stripe spacing saturates, with the excessive holes overflowing into insulating regions , signifying the crossover to conventional metallic phase with overall homogeneity. Besides, the stripes are dynamically fluctuating and may coexist with superconductivity. Dated back to the late 1980's, the relevance of phase separation and dormain walls to high T_c cuprates was already under considerable discussions [5]. In 1993, Emery and Kivelson suggested a scenario of mesoscopic phase separation frustrated by long-range Coulomb interactions [6], as a general consequence of doping a strongly correlated insulator, and they also pointed out the relevance of dynamical stripes to high T_c superconductivity [7]. The origin of phase separation is still under hot debate. Another equally important issue that will be treated here is: assuming the presence of stripes coupled to an undoped background, can we improve our understanding of the interesting and even puzzling physical features revealed in both normal state and superconducting state of cuprates? The importance of such exploration has been recently emphasized in [8] [9], and some interesting results have been reported . Most of these attempts treat the stripe as a 1D or quasi-1D Luttinger Liquid coupled to neighbouring stripes [10] or insulating background which is either modeled as a canonical antiferromagnet [11] or as another 1D Luttinger Liquid [9]. The couplings through pair tunneling [9] or spin exchange [11] have been discussed. Here, in contrast to [9] and [11], I emphasize the importance of coupling a stripe to a truly anomalous 2D insulating background, which has its hidden unconventional nature of RVB (Resonating Valence Bond) spin liquid, under the classical apparel of antiferromagnetic (AFM) order. It is shown that , through single particle hopping [12] between 1D stripes and 2D RVB background, a normal state pseudo-gap Δ_n is "induced" inside the stripe's spin sector, which coincides with the mechanism of spin gap formation in a class of 1D electron systems named after Luther and Emery [13] . Further more, inside the superconducting state, the presence of global phase coherence in RVB order parameter contributes to an even more relevant pairing coupling to 1D stripe, which results in a gap of Δ_{sc} opening in both spin and charge sectors. Experimental consequences of 2 quantitatively different gaps are discussed, and good agreement is found with ARPES results [14].

The brilliant idea of RVB states was advanced by Anderson soon after the discovery of high T_c superconductivity [15]. The RVB state is described by a coherent superstition of different configurations of valence bonds, which was expected to be a reasonable approximation to the ground state of insulating spin 1/2 Heisenberg Model, especially with frustration or hole doping, although the ground state of undoped cuprates clearly has a Neel order. Lately there has been renewed interest in the plausible relevance of RVB correlation to cuprate physics at relatively high energy scale, motivated both experimentally and theoretically. Recent ARPES result on $Ca_2CuO_2Cl_2$ by F. Ronning et al. [16] reveals the presence of a d-wave dispersion along the remnant Fermi surface and a Dirac like dispersion isotropically focused around $(\pi/2, \pi/2)$, which is exactly what was predicted for the "π-flux phase" [21] of RVB spin liquid, where $\epsilon(k) = J\sqrt{\cos^2 k_x + \cos^2 k_y}$, and can not be described within the spin density wave picture although the latter can account for the low-lying spin excitations in the Neel state. Further numerical results also support the presence of a local RVB spin liquid state around a doped hole with momentum $\mathbf{k} = (\pi, \mathbf{0})$ [17], accompanied by an anti-phase of spins around the hole which may be

relevant to the generation of anti-phase domain walls in striped phase of cuprates. Theoretically, Kim and Lee show that Neel order can be restored in π-flux phase through dynamical mass generation of gauge fluctuations at low temperature [18] , which points toward an emerging consistent RVB picture spanning from ground state to high energy scale physics [19]. Based on the above results, I suggest that one can model the environment of a quarter-filled stripe as a RVB spin liquid, which is coupled with the stripe by a single particle hopping term that conserves the momentum along the stripe direction . To get started, one can first ignore the inter-stripe correlation in the normal state of underdoped cuprates, considering the strong incoherence revealed by experiments. The total Hamiltonian is given by

$$H(c,c^+,d,d^+) = H_{1D}(d,d^+) + H_{RVB}(c,c^+) + H_{couple}(c,c^+,d,d^+), \tag{1}$$

where c, c^+ and d, d^+ represent the annihilation and creation operators of a single particle in 2D RVB background and 1D stripe, respectively. $H_{couple}(c,c^+,d,d^+) = \sum_{\mathbf{k},q,\sigma} V c^+_{\mathbf{k},\sigma} d_{q,\sigma}\delta_{k_x,q} + h.c.$, where only horizontal stripe is considered, $\mathbf{k} = (k_x, k_y)$, and momentum conservation is ensured by requiring $k_x = q$. V gives the hopping matrix element, which is vital in deciding different energy scales relevant to cuprates , as will be discussed later [12].

A routine Hartree-Fork decoupling is applied to the RVB Hamiltonian H_{RVB} [15],

$$\begin{aligned} H_{RVB} &= -J\sum_{<ij>} b^+_{ij} b_{ij} \\ &= -J\sum_{<ij>} (\Delta^*_{ij} b_{ij} + \Delta_{ij} b^+_{ij} - |\Delta_{ij}|^2), \end{aligned} \tag{2}$$

where $b^+_{ij} = \frac{1}{\sqrt{2}}[c^+_{i,\uparrow}c^+_{j,\downarrow} - c^+_{i,\downarrow}c^+_{j,\uparrow}]$, Δ_{ij} is the RVB order parameter defined on each bond between 2 nearest neighbors, which is reduced to the mean field average of b_{ij} operator at the saddle point level. Then one can change into the momentum space, that is

$$H_{RVB} = \frac{-J}{\sqrt{2V}}\sum_{\mathbf{k},\mathbf{q}}[\Delta^*_{\mathbf{k},\mathbf{q}}(c_{\mathbf{q},\downarrow}c_{\mathbf{k}-\mathbf{q},\uparrow} - c_{\mathbf{q},\uparrow}c_{\mathbf{k}-\mathbf{q},\downarrow}) + h.c.], \tag{3}$$

where
$\Delta_{\mathbf{k},\mathbf{q}} = \sum_{\hat{\delta}} \Delta_{\mathbf{k},\hat{\delta}} e^{i\mathbf{q}\hat{\delta}}$, and $\Delta_{\mathbf{k},\hat{\delta}} = \frac{1}{\sqrt{V}}\sum_{\mathbf{r_i}} \Delta_{i,i+\hat{\delta}} e^{-i\mathbf{k}\mathbf{r_i}}$ ($\widehat{\delta} = \pm\widehat{\mathbf{x}}, \pm\widehat{\mathbf{y}}$).

There are many possible mean field states in RVB theory [22], among which the "π-flux phase" is selected here, because of its low energy, conservation of time-reversal symmetry and possible connection to AFM long range order [18]. In "π-flux phase", $\Delta_{ij} = \Delta_0 e^{i\phi_{ij}}$ is chosen to have uniform amplitude Δ_0, while its phase ϕ_{ij} is selected to ensure that staggered $+\pi$ and $-\pi$ flux is threaded through each plaquette. For convenience, one can choose $\phi_{ij} = \pm\pi/4$. Therefore, H_{RVB} is simplified to

$$\begin{aligned} H_{RVB} &= -J\sum_{\mathbf{q}}[\Delta^*_0\gamma(\mathbf{q})(c_{\mathbf{q},\downarrow}c_{-\mathbf{q},\uparrow} - c_{\mathbf{q},\uparrow}c_{-\mathbf{q},\downarrow}) + h.c.] \\ &+ iJ\sum_{\mathbf{q}}[\Delta^*_0\eta(\mathbf{q})(c_{\mathbf{q},\downarrow}c_{\hat{\pi}-\mathbf{q},\uparrow} - c_{\mathbf{q},\uparrow}c_{\hat{\pi}-\mathbf{q},\downarrow}) - h.c.], \end{aligned} \tag{4}$$

where $\gamma(\mathbf{q}) = \cos q_x + \cos q_y$, $\eta(\mathbf{q}) = \cos q_x - \cos q_y$ and $\widehat{\pi} = (\pi,\pi)$. Then perform Euclidean path integral over the 2D degrees of freedom and obtain the low energy effective action for 1D stripe as follows

$$\begin{aligned} e^{-S_{eff}} &= \exp\{-\int_0^\beta H_{eff}(d,d^+)d\tau + \frac{1}{\beta}\sum_n i\omega_n d^+ d\} \\ &= \int dcdc^+ \exp\{-\int_0^\beta [H_{1D}(d,d^+) + H_{couple}(c,c^+,d,d^+) \\ &+ H_{RVB}(c,c^+)]d\tau + \frac{1}{\beta}\sum_n i\omega_n(c^+c + d^+d)\}, \end{aligned} \tag{5}$$

where $\omega_n = \frac{\pi n}{\beta}$ (n is odd integer), $\beta = \frac{1}{k_B T}$.

Assuming $J|\Delta_0| >> V$, one can change back to 1D coordinate system and get

$$\begin{aligned} H_{eff} &= H_{1D}(d,d^+) - \frac{V^2}{16J}\sum_l[\frac{1}{\Delta^*_0}(d_{l,\downarrow}d_{l+1,\uparrow} - d_{l,\uparrow}d_{l+1,\downarrow}) \\ &+ h.c.] - i\frac{V^2}{16J}\sum_l[\frac{(-1)^l}{\Delta^*_0}(d_{l,\downarrow}d_{l+1,\uparrow} - d_{l,\uparrow}d_{l+1,\downarrow}) - h.c.]. \end{aligned} \tag{6}$$

Then go to the continuum limit, $d_{l,\sigma} \to \sqrt{a}\Psi(x = la)_\sigma$, with the size of unit cell $a \to 0$ and retain only the slow varying part of H_{eff}, we finally arrive at the following correction to H_{1D} due to its coupling to RVB background,

$$\Delta H_{eff} = g\int dx\Psi_\downarrow(x)\Psi_\uparrow(x) + h.c., \tag{7}$$

where $g = -\frac{V^2\cos k_F}{8J\Delta^*_0}$, and $k_F = \pi/4$ for quarter-filled stripe. We note that the 1D anomalous propagator is induced in stripes through hopping V by the strong pairing correlation inherent to the RVB background. This mechanism is central to the pairing process among mobile carriers inside stripes, via which superconductivity becomes viable.

Based on the above result, I will discuss both normal state and superconducting state , respectively. Let us first come to the issue of normal state pseudo-gap , which is deemed as very important but remains controversial. Theorists are sharply divided in whether it is precursor pairing or otherwise has nothing to do with pairing, but caused by proximity to quantum critical point of , for example AFM phase transition. To treat normal state here, one can take the strong phase fluctuation in RVB order parameter into account , while its amplitude is basically

non-zero and much less fluctuating. Therefore, one can integrate out the phase of Δ_0, and get

$$H_{eff} = H_{1D}(d, d^+) + g_1 \int dx \Psi_\uparrow^+ \Psi_\uparrow \Psi_\downarrow^+ \Psi_\downarrow,$$

where $g_1 \approx \frac{-g^2 a^2}{2v}$, and v is the bare Fermi velocity, which can be treated with the standard bosonization technique [23]as follows

$$\begin{aligned} H_{eff} = \int dx \{ [(\frac{K_c u_c}{2} - \frac{g_1}{2\pi})\Pi_c^2 + \frac{u_c}{2K_c}(\partial_x \Phi_c)^2] \\ + [(\frac{u_s}{2} + \frac{g_1}{2\pi})\Pi_s^2 + (\frac{u_s}{2} + \frac{g_1}{2\pi})(\partial_x \Phi_s)^2] \\ + g_1 \cos(\sqrt{8\pi}\Phi_s) \}, \end{aligned} \quad (8)$$

where Φ_c, Π_c, and Φ_s, Π_s , are conjugated boson operators representing density fluctuations in charge and spin sectors of 1D Luttinger Liquid [23], respectively. u_c, u_s are the corresponding propagating velocities, and K_c is a parameter of interaction.

In terms of renormalization group formulation, $g_1 \cos(\sqrt{8\pi}\Phi_s)$ in H_{eff} is marginally relevant, which results in the opening of a spectral gap in spin sector

$$\Delta_s \propto \sqrt{|g_1|} \exp(\frac{v}{2\pi g_1}),$$

where v is the bare Fermi velocity. Δ_s is here identified as the normal state pseudo-gap Δ_n that leads to spectral weight depletion in low energy spin fluctuations and single particle spectrum, while the charge excitations remain gapless, which give rise to metallic transporting along the stripe. This is of the same principle as the early results by Luther and Emery in exploration of spin gap formation as an instability of Luttinger Liquid [13]. Further more, the effect of g_1 on charge sector is also physically important, it leads to $K_c > 1$ [24], which ensures that singlet superconducting fluctuation dominates over CDW (charge density wave) correlation, and drives the system close to the opening of charge gap and superconducting phase transition that accompanies it (as will be clarified later).

Now let's turn to the superconducting state. It is generally agreed that global phase coherence is established at $T < T_c$, so that strong phase fluctuation in $\Delta_{ij} = \Delta_0 e^{i\phi_{ij}}$ is quenched, and the relevant correction to H_{1D} becomes Eq[7] itself with Δ_0 replaced by its average magnitude.

Then standard bosonization gives

$$\Delta H_{eff} = 2gd_u \int \cos(\sqrt{2\pi}\Theta_c)\sin(\sqrt{2\pi}\Phi_s)dx,$$

The scaling dimension of ΔH_{eff} is $\frac{1}{2} + \frac{1}{2K_c}$, so it is generally relevant except for very strong repulsive interactions(i.e. $K_c < 1/3$). Unlike the normal state case discussed before, in ΔH_{eff} both spin sector and charge sector are coupled together by a relevant effective interaction and spin-charge separation typical of a Luttinger Liquid is thus broken and this kind of " spin-charge recombination " may be relevant to the generation of well-defined quasi-particles in superconducting state [25]. Under scaling to lower energy, $2gd_u$ is renormalized to divergence, so Θ_c and Φ_s oscillate around stable equilibrium positions and gaps open in both spin and charge excitations, which leads to non-magnetic ground state dominated by singlet superconducting fluctuations. For clarity, let's discuss the special case of $K_c = 1$ and $u_s = u_c$. Then H_{1D} can be decoupled into 2 independent Sine-Gordon models of $\Phi_\pm = \frac{1}{\sqrt{2}}(\Theta_c \pm \Phi_s)$, corresponding to 2 branches of free massive fermions. In this case, both spin gap and charge gap are equal, that is

$$\Delta_c = \Delta_s \propto 2\pi|g|d_u \propto \frac{V^2}{J\Delta_0}.$$

In general, the effect of small $|K_C - 1| > 0$ is only to mix the above two branches together, while the qualitative picture of gap formation remains robust. Further more, at leading order , it is expected that $\Delta_{s,c} \propto \frac{V^2}{J\Delta_0} \propto |g_1|^{1/2}$ is a fairly good approximation to start with [25], . One can associate this gap with the superconducting gap Δ_{sc}, identified as the quasiparticle gap measured for example by ARPES in superconducting state.

Provided with two quantitatively different energy scales Δ_n and Δ_{sc} derived above , one can explore their experimental consequences. It is emphasized that, without considering the inter-stripe coherent couplings, V represents the strength of local hopping between a single stripe and its insulating background (its range is limited by inter-stripe distance), through which the strong pairing interaction intrinsic to RVB spin liquid is "transfered" into the stripe, and leads to gap openings in both normal state and superconducting state. In going toward overdoped region, because RVB correlation is significantly suppressed, the relevant energy scale g is reduced to $J\Delta_0$, instead of $V^2/J\Delta_0$ [25]. Because $\frac{\Delta_n}{\Delta_{sc}} \propto \exp(\frac{-v}{\pi a g^2})$, Δ_n is much more suppressed compared with Δ_{sc} [25], which is well consistent with the ARPES results [14], and extensive experimental evidences supporting the "absence" of normal state gap in overdoped region [26]. Besides, by combining the present scenario with the spectral properties of Luther-Emery system [27] , one can understand the broad "edge" feature near $(\pi,0)$ in ARPES of underdoped normal state, as due to the proximity toward charge gap formation that turns the power law singularity ($\propto \omega^{\alpha-1/2}$,$0 < \alpha << 1/2$) into a non-singular edge in $A(k,\omega) \propto \omega^{\alpha-1/2}$ ($\alpha > 1/2$) [25]. However, this singularity is restored in overdoped region where the effect of RVB background on stripe is much weakened, therefore singular peaks with long tails are preserved in $A(k,\omega)$ spectrum, as is consistent with what was observed in ARPES [28] .

In superconducting state, global phase coherence al-

lows single particle hopping between adjacent stripes through higher order process. From the calculation of corresponding matrix element $t' \approx \frac{\hbar^2}{2m^* d^2}$($d$ is inter-stripe distance) , one can extract the effective mass $1/m^* \propto \frac{V^2}{J\Delta_0}$ (underdoped case), and thus estimate the Josephson coupling energy $E_J \approx \frac{\hbar^2 \rho_s}{2m^* d} \propto \frac{\Delta_{sc}}{d}$, where ρ_s is the superfluid density of a single stripe [25] . In underdoped region, one can attribute superconducting transition to the global phase ordering [29] and therefore $T_c \propto E_J \propto x\Delta_{sc}$, which agrees well with two facts: first, $T_c \propto x$; second, $T_{c,max}$ scales with Δ_{sc} among the cuprates family. In overdoped region, $T_c \propto \Delta_{sc} \propto J\Delta_0$ because d is saturated and a new energy scale $J\Delta_0$ takes the place of $\frac{V^2}{J\Delta_0}$, this is consistent with the BCS like relation observed in overdoped cuprates .

Before end, three comments are in order. First, the present scenario opens new route toward the understanding of the subtle relation between pseudo-gap and superconducting gap, in that both Δ_n and Δ_{sc} have the same origin : strong pairing interaction in RVB background, but can be quantitatively different in their dependences on V and $J\Delta_0$. Secondly, one can unify the important energy scales : Δ_n, Δ_{sc}, E_J, T_c, by determining their unique dependences on a single parameter ($V^2/J\Delta_0$ in underdoped region and $J\Delta_0$ in overdoped region), this explains the material-independent scaling in Δ_{sc} : Δ_n : $T_{c,max}$ among cuprates family, while a single material-independent J can not. Thirdly, one can treat the "heavy mass" issue raised recently in [32] within the present picture: in underdoped cuprates, $\frac{k_B T_c}{x} = \hbar v^* \propto \Delta_{sc} \propto V^2/J\Delta_0$ and is roughly doping-independent. It can be connected to the flat dispersion perpendicular to horizontal stripes (Γ to $(0,\pi)$ direction), as suggested in [32], and can be attributed to slow hole motion transverse to stripes [25], which limits the achievement of higher T_c.

In conclusion, I model the stripe phase in high T_c cuprates as a single stripe coupled to the RVB spin liquid background by the single particle hopping. In normal state, the strong pairing interaction inherent in RVB state is therefore transfered into the Luttinger stripe and drives it toward Luther-Emery Stripe with spin-gap formation. The establishment of global coherence in superconducting state contributes to a more relevant coupling to the stripe and leads to gap opening in both spin and charge sectors. Physical consequences of the present picture are discussed, and good agreement is found with the available experimental results in ARPES.

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